B.Sc Part I (Subsidiary)

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Cartesian product of sets:-

Definition:– Let X and Y be any two sets. Let (x, y) be ordered pair, then cartesian product of X and Y denoted by $X \times Y$ (read as X cross Y. and defined as $X \times Y = \{(x, y) : x \in X, y \in Y\}$ i.e. the first element of ordered pair belongs to the first set and second element of ordered pair belongs to the second set.

Theorem:- Show that $A \subseteq B \Rightarrow A \times A = (A \times B) \cap (B \times A)$

Proof:– ~~(i)~~ We have

$$A \subseteq B \Rightarrow A \cap B = A \quad \text{——— (i)}$$

Now,

$$A \times A = (A \cap B) \times (A \cap B) \quad \text{from (i)}$$

$$= (A \cap B) \times (B \cap A)$$

(by Commutative law)

But

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

$$= (A \cap C) \times (B \cap D)$$

$$= (A \times B) \cap (C \times D)$$

replace C by B and D by A, we get

$$(A \cap B) \times (B \cap A) = (A \times B) \cap (B \times A)$$

$$\Rightarrow A \times A = (A \times B) \cap (B \times A)$$

(using (i) for $A \cap B = A$)

Proved